

Flavor Symmetry as a Spontaneously Broken Discrete Permutation Symmetry Embedded in Color

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Abstract

A new mechanism for breaking an internal symmetry spontaneously is discussed, which is intermediate between the Nambu-Goldstone and Wigner modes of symmetry breaking. Here the $q\bar{q}$ sea takes the role of the vacuum of the Nambu-Goldstone case. Flavor symmetry becomes a discrete permutation symmetry of the valence quarks with respect to the sea quarks, which can be spontaneously broken without generation of massless Goldstone bosons.

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It is a well known fact that most hadrons are built from $q\bar{q}$ or qqq valence quarks together with a $q\bar{q}$ and gluon sea. This two-component picture is crucial for my mechanism, to be discussed below, where I shall give the finite $q\bar{q}$ sea a prominent role in the symmetry breaking, which usually is given to the vacuum, when a symmetry is spontaneously broken. I consider flavor symmetry and take $N_f = N_c = 3$ in the discussion, although the same arguments should hold for any number of flavors and colors, and might be applied to other symmetries as well (like chiral symmetry).

If we undress a hadron from its soft confined gluons the $q\bar{q}$ valence quarks of a meson can be thought of as a degenerate nonet in color (and the qqq valence quarks of a baryon as a $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$ multiplet). Likewise the undressed $q\bar{q}$ sea is composed of a nonet and higher representations in color. After dressing with soft gluons, the hadrons

become singlets¹ in color.

Now chose a particular global reference frame in color, in which the sea becomes diagonal, such that it can be composed of diagonal Gell-Mann matrices λ_i : $S(q\bar{q}) = \epsilon_0\lambda_0 + \epsilon_3\lambda_3 + \epsilon_8\lambda_8 = \text{diag}(x, y, z)$. This picks out a special direction and ordering in color space. One can still permute the x, y, z but maintain the diagonal form. This permutation freedom will define my flavor symmetry, and we label one particular choice by the flavors, i.e.,

$$S(q\bar{q}) = \begin{pmatrix} S(u\bar{u}) & 0 & 0 \\ 0 & S(d\bar{d}) & 0 \\ 0 & 0 & S(s\bar{s}) \end{pmatrix} \quad (1)$$

where the diagonal terms need not be equal once flavor symmetry is broken.

Of course, still under global color transformations of both valence and sea one remains within the same hadron, - a π^- remains a π^- and a K^- remains a K^- etc. More precisely the charge and strangeness operators,

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}, \quad S = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2)$$

which define² the charge as $q = \text{Tr}[\Sigma Q \Sigma^\dagger - \Sigma^\dagger Q \Sigma]$ and strangeness $s = \text{Tr}[\Sigma S \Sigma^\dagger - \Sigma^\dagger S \Sigma]$

¹A mechanism by which one can understand this is to assume all gluonic transitions, $q_i\bar{q}_j\text{glue} \rightarrow q_{i'}\bar{q}_{j'}\text{glue}$, between the N degenerate states to be equal $H_{ij,i'j'} = \text{const}$. After diagonalization this gives 0 for all other transitions than singlet to singlet which is $N \cdot \text{const}$, i.e., all other states except the singlet decouple.

²We remind the reader that flavor must be represented by complex or non-Hermitian fields, e.g., $|\pi^+ \rangle = (\lambda_1 - i\lambda_2)/2$, and that charge and strangeness are a kind of interference term between the Hermitian and the anti-Hermitian part. In practice, in a C-invariant theory one can often neglect the anti-Hermitian part and consider instead flavorless, Hermitian superpositions like $|\pi^+ \rangle + |\pi^- \rangle = \lambda_1$.

of a nonet meson Σ , must transform covariantly under color

$$Q' = UQU^\dagger, \quad S' = USU^\dagger, \quad \Sigma' = U\Sigma U^\dagger \quad (3)$$

which of course only implies that our choice of u, d, s above is always done in a particular, but arbitrary color reference frame, as chosen above for convenience

We can write for e.g. the vector flavor nonet when unmixed

$$V = \begin{vmatrix} (\omega + \rho)/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & (\omega - \rho)/\sqrt{2} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{vmatrix} \otimes \begin{vmatrix} S(u\bar{u}) & 0 & 0 \\ 0 & S(d\bar{d}) & 0 \\ 0 & 0 & S(s\bar{s}) \end{vmatrix} \quad (4)$$

Each meson is also a nonet in color (before the gluon dressing), i.e., we have 9 nonets in all. Flavor is a relative quantum number given by the ordering of u, d, s in the valence part with respect to the ordering of the diagonal terms in the sea. Thus one transforms a π^- to a K^- by permuting $d \rightarrow s$ in the valence part but not in the sea. On the other hand in a global color transformation one performs an SU3 rotation (or a permutation of quark labels) in both the valence and the sea part of the wave function. In the limit of a symmetric sea, $S(u\bar{u}) = S(d\bar{d}) = S(s\bar{s})$, all 9 degenerate nonets lie on top of each others, and both flavor and color is unbroken. But if the sea is asymmetric the flavor nonets are generally³ split, but color remains always exact.

A natural mechanism for generating the asymmetric sea is given by quantum loops such as $K^* \rightarrow K^*\pi, K\pi, K\phi, \dots \rightarrow K^*$ or $\pi \rightarrow \pi\sigma, K^*\bar{K}, \dots \rightarrow \pi$ etc. Hadrons are, in fact, unique

³A tricky point is that the sea can be somewhat asymmetric (since $\mathbf{8} \otimes \mathbf{8}$ contains an octet), but still one can have a flavor symmetric spectrum. Thus e.g. all of the d/u asymmetry seen in deep inelastic scattering on the proton need not result in isospin breaking. Only if the d/u asymmetry in the proton is not equal to the u/d asymmetry in the neutron do we have isospin violation. For the s/d quark asymmetry the flavor symmetry breaking is more obvious if one knows that in the sea of all hadrons the s quark is less frequent than the d .

compared to other bound states, like atoms or nuclei, in that they are partly composed of the hadrons themselves, although the latter are in virtual off shell states. A proton is part of the time a proton and a pion, and a pion is part of the time in a three pion state etc. Constituent quarks are again composed of virtual quarks and gluons

$$|q > \propto |q > (1 + \alpha|q\bar{q} > + \beta|q\bar{q}q\bar{q} > + \dots) \times \textit{gluons} \quad (5)$$

i.e., the same constituents occur on both the l.h.s. and the r.h.s. Introducing a reference frame for the quarks, the same reference frame appear on both sides of the equation. A color rotation is like rotating an object (valence) and the observer (sea) resulting in no change for the observer. On the other hand a change (permutation) in part of a state (the valence part) while the rest remains intact results in a new (flavor) state.

Since a strange-antistrange (K^+K^-) virtual state should be less frequent than a non strange ($\pi^+\pi^-$) virtual state loops naturally lead to an asymmetric sea in all hadrons, where $s\bar{s}$ is less frequent than $u\bar{u}$ or $d\bar{d}$. Furthermore, this mechanism can be self-enhancing resulting in an instability: A small initial strange-nonstrange splitting for the (input) virtual states generates a bigger splitting in the output physical state.

Although our ansatz that flavor symmetry is broken by an asymmetric sea, rather than by the infinite vacuum might seem to be a minor one, it has dramatic consequences.

The most important one is that the symmetry can be broken spontaneously, without the appearance of massless Goldstone bosons. This has been a major stumbling block in previous attempts to break flavor symmetry spontaneously. The discrete flavor symmetry introduced above can thus be broken spontaneously without the necessary appearance of unseen Goldstone bosons, and the continuous SU3 symmetry is never broken. In a series of previous publications [1] actual dynamic calculations of such spontaneous symmetry breaking was preformed. These involved only scalars in a simplified model, and pseudoscalars and vectors in a little more realistic model. It was demonstrated that the symmetry breaking goes in the right direction compared to experiment, and that e.g. the mechanism naturally explains the approximate nature of the Okubo-Zweig-Iizuka rule, the near ideal mixing and

the equal spacing rule of meson multiplets.

Furthermore this opens up a new scenario for predicting quark masses (and possibly the CKM matrix), since one can start from an exactly symmetric theory with few parameters (and in accord with the flavor blindness of QCD) which can have both an unstable symmetric and a stable flavor asymmetric solution. Then, of course, only the stable asymmetric one is the true physical solution.

Of course our discrete symmetry can also be broken explicitly. In fact, one expects that electro-weak interactions should break the up-down symmetry and give the u, c, t quarks an extra mass, while the d, s, b quarks may be degenerate or nearly massless before the spontaneous breaking by strong interactions. A true lepton-quark symmetry might emerge.

There is some similarity with the color-flavor connection discussed here, and the color-flavor locking of Schäfer and Wilczek [2] and collaborators, although the latter is applied within another context of high density QCD, where the $q\bar{q}$ sea and the vacuum merge.

The suggested new interpretation of flavor symmetry also throws some new light on the nature of superselection rules [3], which was vigorously discussed almost half a century ago when isospin invariance had been introduced.

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